Mass relation of ρ and a_1 mesons

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Abstract

A mass relation of ρ and a_1 mesons has been obtained from Weinberg's first sum rule and KSFR sum rule.

One of the most important features revealed from quantum chromodynamics (QCD) is the chiral symmetry in the limit of $m_q \to 0$ $(q = u, d, s)$. On the other hand, current algebra is a successful theory and Vector Meson Dominance(VMD) is fruitful in studying electromagnetic properties of hadrons. We abbreviate chiral symmetry, current algebra, and VMD as CHCV. CHCV are powerful tools used to study hadron physics at low energies(nonperturbative QCD). Effective chiral theory of mesons[1] is one of important approaches used to study nonperturbative QCD. CHCV constrain the structure of effective chiral theory of mesons. ρ and a_1 mesons have been treated as chiral partners. Weinberg's first sum rule[2] has been established by CHCV. By invoking an additional assumption, Weinberg's second sum rule[2]

$$g_\rho = g_a \tag{1}$$

has been found. The mass relation

$$m_a^2 = 2m_\rho^2 \tag{2}$$

is the consequence of eq.(1) and the KSFR sum rule[3]

$$g_\rho^2 = \frac{1}{2} F_\pi^2 m_\rho^2. \tag{3}$$

The mass formula(eq.(2)) predicts that $m_a = 1089MeV$ and the data[4] shows $m_a = 1230 \pm 40MeV$. In this letter we try to improve the mass relation(eq.(2)) based on CHCV only.

By using CHCV, Weinberg's first sum rule[2]

$$\frac{g_\rho^2}{m_\rho^2} - \frac{g_a^2}{m_a^2} = \frac{F_\pi^2}{4} \tag{4}$$

has been derived, where

$$< 0|V_\mu^a|\rho_b^\lambda >= \epsilon_\mu^\lambda \delta_{ab} g_\rho,$$

$$< 0|A_\mu^a|a_b^\lambda >= \epsilon_\mu^\lambda \delta_{ab} g_a,$$

and F_π is pion decay constant, $F_\pi = 186MeV$. This sum rule(eq.(4)) can be tested. g_ρ is the coupling constant between ρ and γ and it has been determined to be

$$g_\rho = 0.116(1 \pm 0.05)GeV^2 \tag{5}$$

from $\Gamma(\rho \to l^+l^-)$. Using $m_a = (1230 \pm 40)MeV$, the eq.(4) predicts that

$$g_a = 0.145 \pm 0.018GeV^2. \tag{6}$$

From eqs.(5,6) it can be seen that Weinberg's second sum rule(eq.(1)) is not in good agreement with data. On the other hand, g_a can be determined from

$$\Gamma(\tau \to a_1\nu) = \frac{G^2}{8\pi}cos^2\theta_c g_a^2 \frac{m_\tau^3}{m_a^2}(1 - \frac{m_a^2}{m_\tau^2})^2(1 + 2\frac{m_a^2}{m_\tau^2}). \tag{7}$$

The experimental data of the decay rate is[4] $2.14 \times 10^{-13}(1 \pm 0.32)GeV$ and g_a is determined to be $0.145 \pm 0.033GeV^2$. Therefore, Weinberg's first sum rule is in good agreement with the data. On the other hand, the KSFR sum rule(eq.(3)) agrees with data within 10%. The combination of eqs.(3,4) leads to a new mass relation between ρ and a_1. In order to see that let's rewrite eq.(4) as

$$1 - \frac{g_a^2}{g_\rho^2}\frac{m_\rho^2}{m_a^2} = \frac{F_\pi^2}{4}\frac{m_\rho^2}{g_\rho^2} \tag{8}$$

Substituting KSFR sum rule into eq.(8) we obtain

$$m_a^2 = 2\frac{g_a^2}{g_\rho^2}m_\rho^2 \tag{9}$$

Comparing with eq.(2) there is an additional factor of $\frac{g_a^2}{g_\rho^2}$ in eq.(9) and there is no any additional assumption. Using the values of g_a and g_ρ(eqs.(5,6)) we obtain

$$m_a = 1.36(1 \pm 0.17)GeV \tag{10}$$

Theoretical prediction of eq.(9) fits the data of m_a better.

To conclude, a new mass relation between ρ and a_1 mesons has been presented in terms of Weinberg's first sum rule and KSFR sum rule.

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References

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